Mechanics

Forces

3B PHYSICS EXPERIMENT

Parallelogram of forces

EXPERIMENTAL INVESTIGATION OF THE VECTOR ADDITION OF FORCES

- Plotting the equilibrium of three arbitrary forces on a graph
- Analytical investigation of the point of equilibrium when forces F_1 and F_2 are symmetrical

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BASIC PRINCIPLES

Forces are vectors and can therefore be added using the rules of vector addition. To demonstrate the sum of two vectors on a graph, the point of origin of the second vector is placed on the final point of the first vector. The arrow from the point of origin of the first vector to the final point of the second vector represents the resultant vector. By completing the parallelogram (of which the two vector lines are sides), a diagonal drawn from the original angle to the opposite corner represents the resultant vector (see fig. 1).
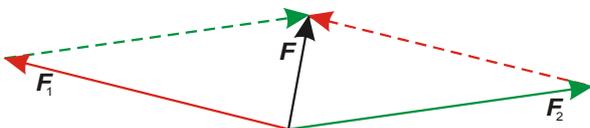

Fig. 1: Vector sum of forces (parallelogram of forces)

The vector addition of forces can be demonstrated in a clear and simple manner on the force table. The point of action of three individual forces in equilibrium is exactly in the middle of the table. Determine the magnitude of the individual forces from the suspended weights and, using a protractor, note the angle of each force vector (the direction of each force).

In a state of equilibrium, the sum of the three individual forces is given by:

$$(1) \quad F_1 + F_2 + F_3 = 0$$

F_3 is therefore the sum of individual forces F_1 and F_2 (see fig. 2):

$$(2) \quad -F_3 = F = F_1 + F_2$$

The parallel vector components for sum F are given by:

$$(3) \quad -F_3 = F = F_1 \cdot \cos\alpha_1 + F_2 \cdot \cos\alpha_2$$

and the vertical components are given by:

$$(4) \quad 0 = F_1 \cdot \sin\alpha_1 + F_2 \cdot \sin\alpha_2$$

Equations (3) and (4) provide a mathematical analysis of the vector addition. For the experiment, it is advisable to align force F_3 at an angle of 0°.
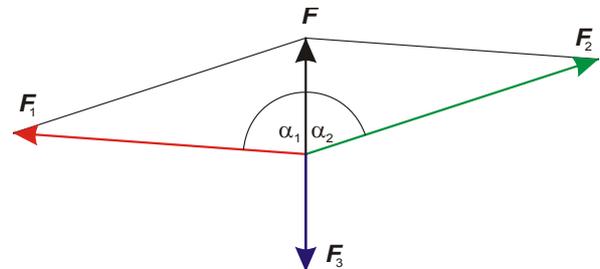

Fig. 2: Determining the sum of vectors of two forces F_1 and F_2 from equilibrium force F_3.

For analytical observations, the equilibrium of forces can alternatively be investigated on a graph. To do so, draw lines representing all three forces diverging from the central point of action. Note the magnitude and angle of each force. Subsequently, displace forces F_2 and F_3 along a parallel path till the point of origin is at the end of the preceding vector. The resultant vector is 0 (see fig. 3). In the experiment, carry out this procedure for three arbitrary forces, making sure to maintain the state of equilibrium every time.
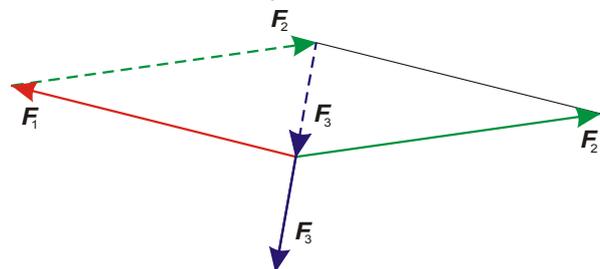

Fig. 3: Graphic investigation of the equilibrium of three arbitrary forces acting in different directions

In the experiment, the analytical observation is restricted to the special situation that the two forces F_1 and F_2 are symmetric to F_3.

LIST OF APPARATUS

1   Force Table   U52004   1000694

SET-UP

Fig. 4: Experiment set-up

- Set up the force table on a level surface.
- Attach the pulleys for the three force components at 60°, 180° and 300°.
- Attach three strings to the white ring using the fixing clips, pass each one over a pulley and load with a full set of slotted weights.
- Check whether the white ring is positioned symmetrically relative to the centre of the table.
- If necessary, correct the alignment of the table and of the strings.

EXPERIMENT PROCEDURE

a) With F_1 and F_2 in a symmetrical configuration:

- Keep force component F_3 at 180° throughout.
- Set up components F_1 and F_2 at 10° (angle α_1 for F_1) and 350° (-10°) and apply 100 g load to each.
- Adjust the loading of the F_3 component until the white ring is in the equilibrium position and record the relevant value of the suspended mass m_3 in table 1.
- Set up the force components F_1 and F_2 at 20° and 340° (-20°) and change the mass m_3 until the equilibrium is restored.
- Change the angle α_1 to the values 30°, 40°, 50°, 60°, 70° and 90° in turn. In each case determine the mass m_3 needed to restore equilibrium, and record the values in table 1.

b) With force components in arbitrary directions:

- Set the force component F_1 at 340° and apply 50 g load.
- Set the force component F_2 at 80° and apply 70 g load.
- Adjust the orientation and loading of the force component F_3 until the forces are in equilibrium.

SAMPLE MEASUREMENTS

a) With F_1 and F_2 in a symmetrical configuration:

Table 1: Mass m_3 and corresponding (calculated) force F_3 needed to maintain equilibrium between the forces, as a function of the angle α_1 ($m_1 = m_2 = 100$ g, $F_1 = F_2 = 100$ g).

α_1	m_3 (g)	F_3 (N)
10°	200	2.00
20°	190	1.90
30°	170	1.70
40°	155	1.55
50°	130	1.30
60°	200	2.00
70°	70	0.70
90°	0	0.00

b) With force components in arbitrary directions:

Table 2: Angles α_i of the force components, load masses m_i and corresponding (calculated) forces F_i

α_1	m_1 (g)	F_1 (N)	α_2	m_2 (g)	F_2 (N)	α_3	m_3 (g)	F_3 (N)
350°	50	0.5	80°	70	0.7	221°	80	0.8

EVALUATION

a) With F_1 and F_2 in a symmetrical configuration:

Equation (4) is satisfied in a symmetric case ($F_1 = F_2$ and $\alpha_1 = -\alpha_2$. From equation (3) we get the characteristic equation used for the vector sum

$$F = 2 \cdot F_1 \cdot \cos \alpha_1 .$$

A calculation using this equation gives the curve shown in figure 5, which agrees with the experimental data in table 1 within the limits of accuracy of the measurements.
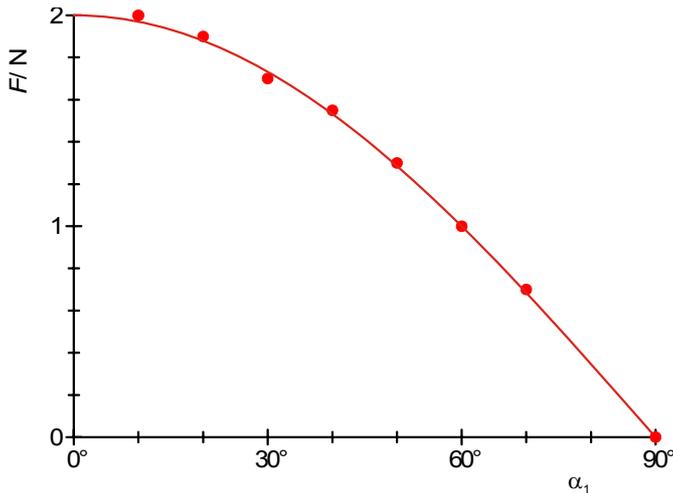

Fig. 5: Measured and calculated sums of two symmetric forces in relation to the angle α_1

b) With force components in arbitrary directions:

For a graphical analysis of the experimental data in table 2, the three forces are first represented as lines (vectors) outward from the central point of action, with their magnitudes and angles. The lines representing the forces F_2 and F_3 are then shifted while keeping their directions fixed, so that the starting point of each vector lies at the end point of the previous vector.

The length of the resulting vector sum is found to be zero, within the limits of accuracy of the measurements.
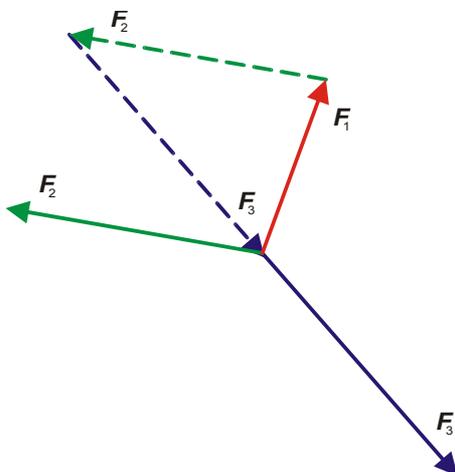

Fig. 6: Vector graphic of forces corresponding to experimental data in table 2 and the resultant of all the forces.