Inclined Launch

PLOT “PARABOLIC” TRAJECTORIES POINT BY POINT

- Measure the distance covered in a trajectory as a function of the angle at which a projectile is launched and the initial velocity.
- Calculate the initial velocity from the maximum distance covered by the trajectory.
- Point-by-point plotting of the “parabolic” trajectory as a function of the launch angle and the initial velocity.
- Verification of the principle of superposition.

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BASIC PRINCIPLES

According to the principle of superposition, the motion of a ball that is propelled upward at an angle to the horizontal in the earth’s gravitational field is the combination of a motion at a constant speed in the direction of launch and a gravitational falling motion. This results in a parabolic flight curve, whose height and width depend on the launch angle α and the initial velocity v_0.

To calculate the theoretical trajectory, take the centre of the spherical ball as the origin of the coordinate system for simplicity and neglect the frictional drag of the air on the ball. Thus the ball retains its initial velocity in the horizontal direction.

$$v_x(0) = v_0 \cdot \cos\alpha \quad (1)$$

Therefore at time *t* the horizontal distance covered is

$$x(t) = v_0 \cdot \cos\alpha \cdot t \quad (2)$$

In the vertical direction, under the influence of the gravitational, field the ball is subjected to gravitational acceleration *g*. Therefore, at time *t* its vertical velocity is

$$v_y(t) = v_0 \cdot \sin\alpha - g \cdot t \quad (3)$$

The vertical distance travelled is

$$y(t) = v_0 \cdot \sin\alpha \cdot t - \frac{1}{2} \cdot g \cdot t^2 \quad (4).$$

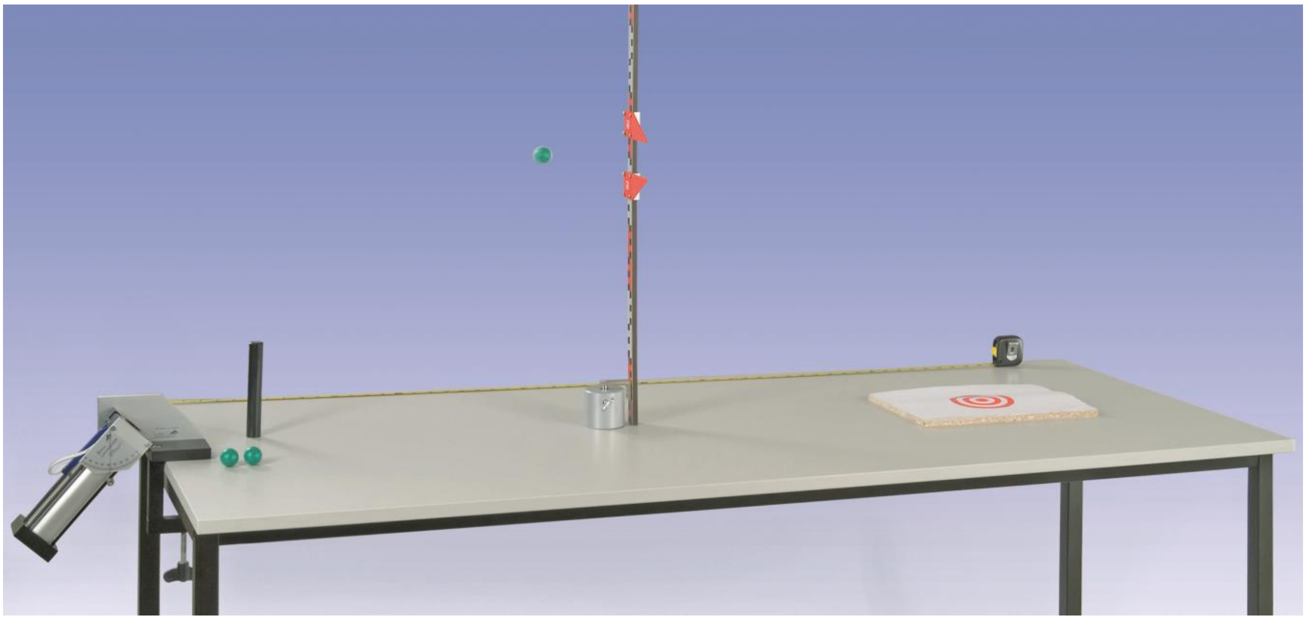

Fig. 1: Set-up for point-by-point measurement of parabolic trajectories

LIST OF APPARATUS

1 Projectile Launcher	1002654 (U10360)
1 Clamp for Projectile Launcher	1002655 (U10361)
1 Vertical Ruler, 1 m	1000743 (U8401560)
1 Set of Riders for Rulers	1006494 (U8401570)
1 Barrel Foot, 1 kg	1002834 (U13265)
1 Pocket Measuring Tape, 2 m	1002603 (U10073)

The trajectory of the ball has the form of a parabola, as it conforms to the equation

$$y(x) = \tan\alpha \cdot x - \frac{1}{2} \cdot \frac{g}{(v_0 \cdot \cos\alpha)^2} \cdot x^2 \qquad (5)$$

The ball reaches the highest point of the parabola t time t_1 given by:

$$t_1 = \frac{v_0 \cdot \sin\alpha}{g} \qquad (6).$$

It returns to its initial height 0 at time t_2 given by:

$$t_2 = 2 \cdot \frac{v_0 \cdot \sin\alpha}{g} \qquad (7).$$

Thus, the height of the parabola is

$$h = y(t_1) = \frac{v_0{}^2}{2 \cdot g} \cdot \sin^2\alpha \qquad (8).$$

The distance covered by the ball is thus:

$$s = x(t_2) = 2 \cdot \frac{v_0{}^2}{g} \cdot \sin\alpha \cdot \cos\alpha \qquad (9).$$

In the experiment, the trajectories of a wooden ball are measured point by point as a function of the launch angle and the initial velocity, using a height scale with two pointers (refer to Fig. 3). The horizontal component x of the trajectory is determined from the horizontal distance X to the right-hand edge of the launcher holder measured with a tape:

$$x = X + 110\,\text{mm} \qquad (10).$$

The vertical component y is calculated from the positions Y_1 and Y_2 of the two pointers by assuming that the ball passes exactly through the middle of the gap. The calculation must take into account the fact that the zero level of the height measurements is at the upper edge of the table, whereas the ball starts from a point 37.5 mm above that:

$$y = \frac{Y_2 + Y_1}{2} - 37.5\,\text{mm} \qquad (11).$$

The maximum deviation of the calculated value from the true value is:

$$\Delta y = \frac{Y_2 - Y_1}{2} - 12.5\,\text{mm} \qquad (12).$$

SET-UP

- Clamp the holder for the projectile launcher to the edge of a table with a length of at least 2 m, and mount the launcher as described in the instruction sheet.
- Starting at the right-hand edge of the launcher holder, unroll the measuring tape and fix it to the table.
- Mount the target shown in Figure 2 on a support 25 mm high and lay it some distance away from the projectile launcher.
- Behind that, set up a board to “catch” the ball as it bounces beyond the target.

Fig. 2: Target on which the ball is to land

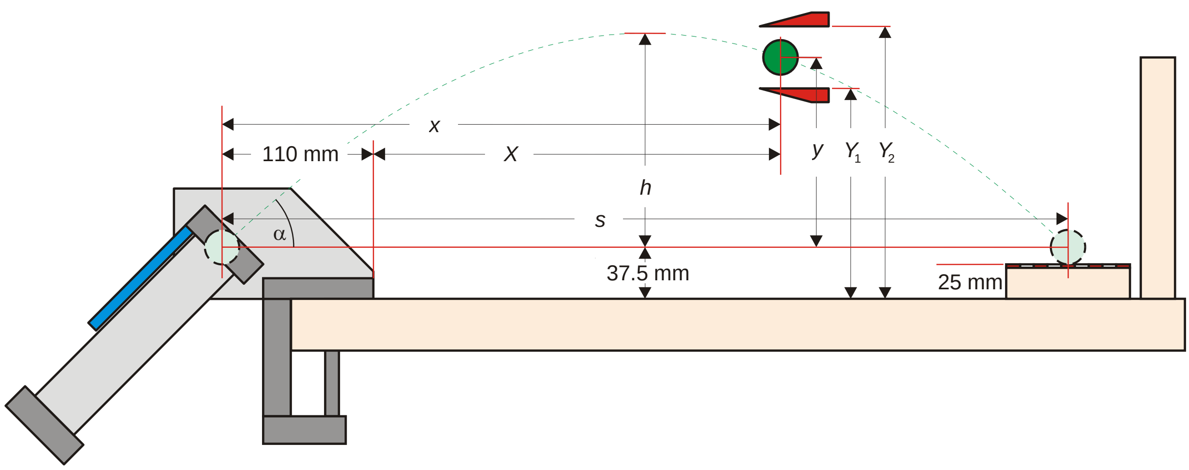

Fig. 3: Schematic diagram for experiment

SAFETY INSTRUCTIONS

Although the kinetic energy of the projected ball is not very high, ensure nevertheless that it cannot hit a person in the eye.

- Never look directly into the projectile's path from the launcher!
- The position of the ball in the launcher should only be checked through the hole at the side.
- Before launching the ball, ensure that nobody is in the flight path.

EXPERIMENT PROCEDURE

Measure the range or distance covered as a function of the launch angle:

- Set the launch angle α at 30°.
- Place the target at a distance of about 1 m.
- Prepare the projectile launcher with its spring tension at the lowest value as described in the instruction sheet.
- Release the ball and observe its trajectory.
- Move the target to the position where the ball landed.
- Repeat the launching of the ball and re-position the target until the ball lands in the centre of the target.
- Measure the distance X to the centre of the target and enter the value in Table 1.
- Make similar measurements with the launch angle α set at 45°, 60° and 75°.
- Using Equation (10), calculate the projectile range s from the values of X in Table 1 and enter them in the table.

Measure the maximum range as a function of the initial velocity:

- Set the launch angle α to 45°.
- Prepare the projectile launcher with its spring tension at the medium value as described in the instruction sheet.
- Release the ball and observe its trajectory.
- Move the target to the position where the ball landed.
- Repeat the launching of the ball and re-position the target until the ball lands in the centre of the target.
- Measure the distance X to the centre of the target and enter the value in Table 2.
- Make similar measurements with the spring tension set at its maximum.
- Using Equation (10), calculate the maximum range s_{max} from the values of X in Table 2 and enter them in the table.

Point-by-point plotting of parabolic trajectory as a function of launch angle:

- Set the launch angle α at 30° and position the target so that with the lowest spring tension the ball lands in the centre of the target.
- Set up a vertical ruler in the stand base and position it at X = 100 mm.
- Set the pair of pointers at Y_1 = 110 mm and Y_2 = 140 mm.
- Launch the ball using minimum spring tension and check whether it travels unhindered and lands in the centre of the target.
- If necessary, correct the positions of the pointers until the ball passes freely and lands in the centre of the target.
- Enter the values of X, Y_1 and Y_2 in Table 3 and from these calculate x, y and Δy.
- Increase the distance X in steps of 50 mm, and each time correct the positions of the pointers until, with the lowest spring tension, the ball lands in the centre of the target.
- Make similar measurements with the launch angle α set at 45°, 60° and 75° and enter the results in Tables 4, 5 and 6.
- Make similar measurements with greater spring tensions, as far as the available experiment space allows.

SAMPLE MEASUREMENTS

Measure the range as a function of the launch angle:

Table 1: Range s as a function of the launch angle with the lowest initial velocity

α	X / mm	s / mm
30°	920	1030
45°	1100	1210
60°	910	1020
75°	465	575

Table 2: Maximum range s_{max} as a function of the initial velocity

Spring tension setting	X / mm	s_{max} / mm	v_0 / m/s
1	1100	1210	3.45
2	2230	2340	4.79
3	4490	4600	6.72

Point-by-point plotting of “parabolic” trajectories as a function of the launch angle:

Table 3: Coordinates of the trajectory with launch angle α = 30°

X / mm	x / mm	Y_1 / mm	Y_2 / mm	y / mm	Δy / mm
100	210	110	140	87.5	2.5
150	260	135	165	112.5	2.5
200	310	155	185	132.5	2.5
250	360	165	195	142.5	2.5
300	410	175	205	152.5	2.5
350	460	180	210	157.5	2.5
400	510	185	215	162.5	2.5
450	560	185	215	162.5	2.5
500	610	180	210	157.5	2,5
550	660	175	205	152.5	2.5
600	710	165	200	145.0	5
650	760	150	185	130.0	5
700	810	140	170	117.5	2.5
750	860	120	155	100.0	5

Table 4: Coordinates of the trajectory with launch angle α = 45°

X / mm	x / mm	Y_1 / mm	Y_2 / mm	y / mm	Δy / mm
0	110	120	155	100	5
50	160	160	195	140	5
100	210	195	225	172,5	2.5
150	260	225	260	205	5
200	310	255	290	235	5
250	360	275	310	255	5
300	410	295	330	275	5
350	460	310	345	290	5
400	510	325	355	302.5	2.5
450	560	330	360	307.5	2.5
500	610	330	360	307.5	2.5
550	660	325	355	302.5	2.5
600	710	320	350	297.5	2.5
650	760	310	340	287.5	2.5
700	810	290	320	267.5	2.5
750	860	270	305	250	5
800	910	245	285	227.5	7.5
850	960	220	255	200	5
900	1010	185	225	167.5	7.5
950	1060	145	190	130	10
1000	1110	110	150	92.5	7.5

Table 5: Coordinates of the trajectory with launch angle α = 60°

X / mm	x / mm	Y_1 / mm	Y_2 / mm	y / mm	Δy / mm
0	11	195	245	182.5	12.5
50	16	260	305	245	10
100	21	310	350	292.5	7.5
150	26	370	410	352.5	7.5
200	31	405	440	385	5
250	36	440	485	425	10
300	41	465	495	442.5	2.5
350	46	480	510	457.5	2.5
400	51	480	510	457.5	2.5
450	56	475	505	452.5	2.5
500	61	460	490	437.5	2.5
550	66	435	470	415	5
600	71	405	445	387.5	7.5
650	76	355	400	340	10
700	81	310	355	295	10
750	86	245	295	232.5	12.5
800	91	170	240	167.5	22.5

Table 6: Coordinates of the trajectory with launch angle α = 75°

X / mm	x / mm	Y_1 / mm	Y_2 / mm	y / mm	Δy / mm
0	110	310	430	332.5	47.5
50	160	450	510	442.5	17.5
100	210	525	570	510	10
150	260	575	605	552.5	2.5
200	310	575	610	555	5
250	360	540	585	525	10
300	410	470	525	460	15
350	460	360	440	362.5	27.5
400	510	225	320	235	35

EVALUATION

Measure the range as a function of the launch angle:

Figure 4 shows how the range *s* depends on the launch angle α as given by the experimental data in Table 1. A theoretical curve fitted to the experimental points was calculated for v_0 = 3.42 m/s using Equation (9).

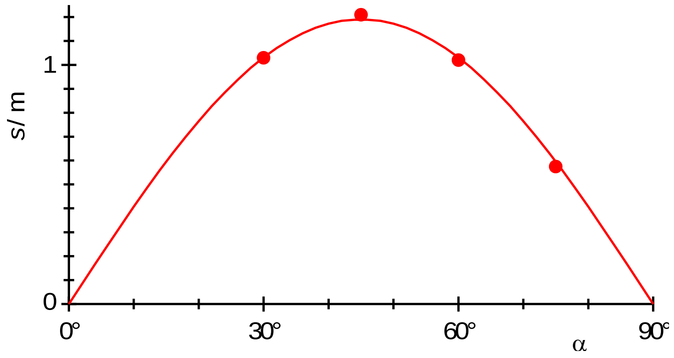

Fig. 4: Distance propelled as a function of the launch angle

The maximum distance covered during any of the trajectories, s_{max}, is reached when the launch angle α is 45°.

Measuring the maximum range s_{max} as a function of the initial velocity v_0:

From the maximum range s_{max} that is achieved with a launch angle of 45°, the initial velocity v_0 can be calculated. From Equation (9) it follows that:

$$v_0 = \sqrt{g \cdot s_{max}}$$

The results of this calculation are given in Table 2.

Point-by-point plotting of “parabolic” trajectories for different launch angles:

The data for the trajectories listed in Tables 3 to 6 are plotted graphically in Figure 5. A precise analysis shows that the trajectories deviate slightly from a parabolic shape because of the frictional drag of air resistance on the ball.

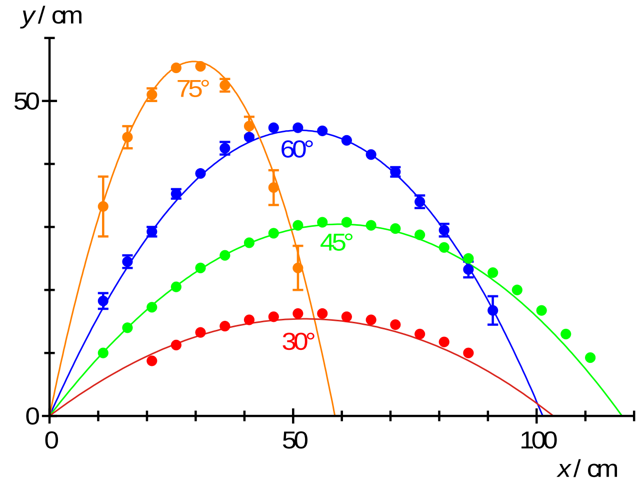

Fig. 5: Trajectories for the smallest initial velocity and different launch angles, measured experimentally (points), and calculated theoretically (curves) with air friction taken into account.