Harmonic Oscillations

MEASURE THE OSCILLATIONS OF A COIL SPRING PENDULUM USING AN ULTRASONIC MOTION SENSOR

- Carry out static determination of spring constants *k* for various coil springs.
- Record the harmonic oscillation of a coil spring pendulum as a function of time using an ultrasonic motion sensor.
- Determine the period of oscillation *T* for various combinations of spring constant *k* and mass *m*.

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Fig. 1: Experiment set-up.

GENERAL PRINCIPLES

Oscillations occur when a system disturbed from its equilibrium position is affected by a force which acts to restore it to equilibrium. This is known as simple harmonic oscillation if the restoring force is proportional to the deviation from the equilibrium position at all times. The oscillations of a coil spring pendulum are one classic example of this. The proportionality between the deviation and the restoring force is described by Hooke's law.

The law states that the relationship between the deviation *x* and the restoring force *F* is given by

(1) $F = -k \cdot x$

where *k* = spring constant

For a weight of mass m suspended from the spring, the following therefore holds:

$$(2)\quad m\cdot\frac{d^2x}{dt^2}+k\cdot x=0\,.$$

This applies as long the mass of the spring itself and any friction that might arise can be neglected.

In general, solutions to this equation of motion take the following form:

$$(3)\quad x(t)=A\cdot\sin\left(\sqrt{\frac{k}{m}}\cdot t+\varphi\right).$$

This will be verified by experiment by recording the harmonic oscillations of a coil spring pendulum as a function of time with the help of an ultrasonic motion sensor and matching the measured data to a sine function.

The ultrasonic motion sensor detects the distance between itself and the weight suspended from the spring. Other than an offset for the zero point, which can be compensated for by calibration, the measurement corresponds directly to the variable $x(t)$ included in equation 3.

The period of oscillation T is defined as the interval between two points where a sine wave crosses the zero axis in the same direction. From equation (3) it can therefore be seen to be equal to:

$$(4)\quad T=2\pi\cdot\sqrt{\frac{m}{k}}\,.$$

In order to verify equation (4), the measurements are made for various combinations of mass m and spring constant k, whereby the period of oscillation is determined from where a curve matching the data crosses the zero axis or by matching the curve to equation (3). The spring constants are also to be established by static measurements and compared with those obtained from dynamic measurements.

LIST OF EQUIPMENT

1	Set of Helical Springs for Hooke's Law	U40816	1003376
1	Set of Slotted Weights, 10x 10 g	U30031	1003227
1	Set of Slotted Weights, 5x 50 g	U30033	1003229
1	Tripod Stand, 150 mm	U13270	1002835
1	Stainless Steel Rod, 1000 mm	U15004	1002936
1	Clamp with Hook	U13252	1002828
1	Ultrasonic Motion Sensor	U11361	1000559
1	3B NET*lab*™	U11310	1000544
1	3B NET*log*™	U11300	1000539/40
1	Pocket Measuring Tape, 2 m	U10073	1002603

EXPERIMENT SET-UP AND PROCEDURE

Note:
The experiment is carried out using spring pendulums involving coil springs with with spring constants specified as k = 2.5, 5 and 25 N/m by way of example.

Static measurement

- Set up the apparatus for the measurement as shown in Fig. 1.
- Suspend one of the Hooke's law springs (nominal values of k = 2.5, 5, 10, 15 and 25 N/m) from the clamp with hook.
- Depending on the stiffness of the spring, add the weights from the 10 x 10 g or 5 x 50 g sets of slotted weights to the spring one after the other, then use the pocket tape measure to find the extension s and enter it into Table 1.

Note:
The weight holders in the slotted weight sets count as one of the ten 10 g or the five 50 g weights.

- Repeat the set of measurements for each of the other springs.

Dynamic measurement

- Set up the apparatus for the measurement as shown in Fig. 1.
- Suspend one of the Hooke's law springs (nominal values of k = 2.5, 5, 10, 15 and 25 N/m) from the clamp with hook.
- Take the four 50 g weights in the 5 x 50 g slotted weight set off the holder. Suspend the holder from the coil spring.
- Place the ultrasonic motion sensor precisely underneath the spring with weight holder hanging from it.
- Connect the ultrasonic motion sensor to analog Input A or B of the 3B NET*log*™ unit using a miniDIN cable. Turn on the 3B NETl*og*™ unit and wait for it to detect the sensor.
- Turn on the computer and start running the 3B NET*l*ab™ software. Connect the 3B NET*log*™ unit to the computer. Configure the input as described in the manual for the 3B NET*lab*™ software.
- Enter the measurement interval/rate, the number of measurements to be made and the duration of the measurement (e.g. 5ms/200Hz for the scope, 1000, 5.0s).
- Slightly deflect the spring pendulum, let it go and simultaneously start the measurement procedure by clicking the "Start" button in the 3B NET*lab*™ software.
- Save the data recorded for the oscillation.
- Add 50 g weights to the holder one by one and repeat the measurement in each case.
- Repeat the whole set of measurements for the other springs.

SAMPLE MEASUREMENT

Static measurement

Tab. 1: Deflection *s* for coil spring with specified spring constant *k* = 2.5 N/m with various masses *m* suspended from it.

m / g	*s* / cm
10	3.2
20	7.2
30	11.2
40	15.4
50	19.7
60	23.7
70	27.7
80	31.7
90	36.0
100	40.0

Tab. 2: Deflection *s* for coil spring with specified spring constant *k* = 5 N/m with various masses *m* suspended from it.

m / g	*s* / cm
10	0.9
20	3.0
30	4.7
40	6.2
50	7.9
60	9.4
70	10.9
80	12.5
90	14.0
100	15.7

Tab. 3: Deflection *s* for coil spring with specified spring constant *k* = 25 N/m with various masses *m* suspended from it

m / g	*s* / cm
50	1.4
100	3.2
150	5.0
200	6.9
250	8.7

Dynamic measurement

Fig. 2 shows the oscillation data recorded by the 3B NET*lab*™ software using the spring with nominal spring constant *k* = 5 N/m carrying a mass *m* = 250 g as an example. In order to verify equation (3), the region of the measured curve between the two cursors is fitted to a sine function.

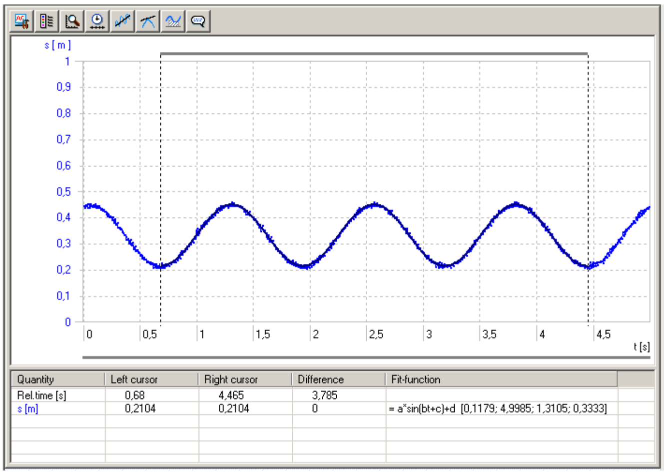

Fig. 2: Recorded oscillation after fitting to a sine function. The cursors mark the two ends of the region to be fitted.

EVALUATION

Static measurement

The weight F_G is equal to the force of the spring F_F, i.e. according to the laws of Newton and Hooke, the following is true:

$$F_G = m \cdot g = k_s \cdot s = F_F \Leftrightarrow s = \frac{g}{k_s} \cdot m = B \cdot m$$

$$B = \frac{g}{k_s} \Leftrightarrow k_s = \frac{g}{B} \quad (5)$$

F_G: Weight
m: Mass of suspended weights
g: Acceleration due to gravity
F_F: Force of spring
k_s: Spring constant
s: Deflection of spring

- Plot the measurement values from tables 1, 2 and 3 n (Fig. 3), and draw a straight line $s = B_s \cdot m$ through the points. Use equation (5) to determine the spring constant k_s from the gradient B_s.

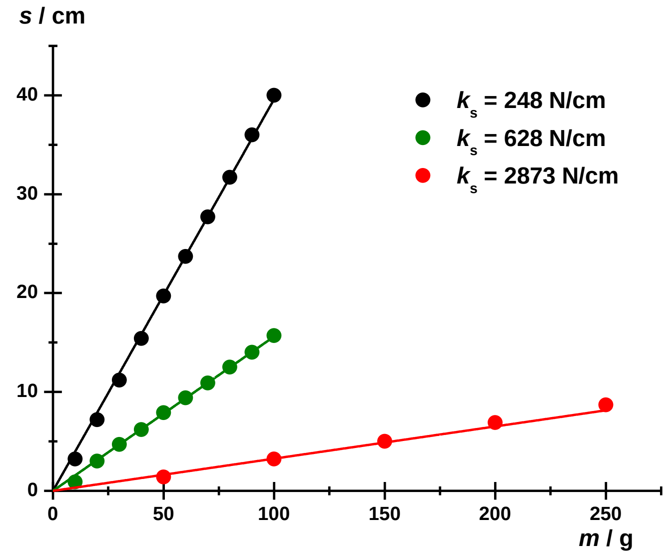

Fig. 3: Deflection s as a function of m.

Dynamic measurement

- Determine the period of oscillation T for each set of oscillation data.
- Do this by reading off the time between every second crossing of the zero axis direct from the curve and entering the results into tables 4, 5 and 6. Alternatively, the period can be determined with the help of equation (4) from the curve as fitted to equation (3).

Tab. 4: Period of oscillation for coil spring with specified nominal spring constant $k = 2.5$ N/m as obtained from recorded oscillation data.

m / g	T / s	T^2 / s^2
50	0.937	0.877
100	1.308	1.710
150	1.503	2.258

Tab. 5: Period of oscillation for coil spring with specified nominal spring constant $k = 5$ N/m as obtained from recorded oscillation data.

m / g	T / s	T^2 / s^2
50	0.584	0.341
100	0.810	0.656
150	0.992	0.983
200	1.143	1.305
250	1.262	1.592

Tab. 6: Period of oscillation for coil spring with specified nominal spring constant $k = 25$ N/m as obtained from recorded oscillation data

m / g	T / s	T^2 / s^2
50	0.289	0.084
100	0.398	0.158
150	0.482	0.232
200	0.553	0.305
250	0.619	0.384

From equation (4):

$$T^2 = \frac{4\pi^2}{k_T} \cdot m = B_T \cdot m \qquad (6)$$
$$B_T = \frac{4\pi^2}{k_T} \Leftrightarrow k_T = \frac{4\pi^2}{B_T}.$$

- Plot the square of the period measurements from tables 4, 5 and 6 (Fig. 4) against the mass and draw a straight line $T^2 = B_T \cdot m$ through the points. Use equation (5) to determine the spring constant k_T from the gradient B_T.

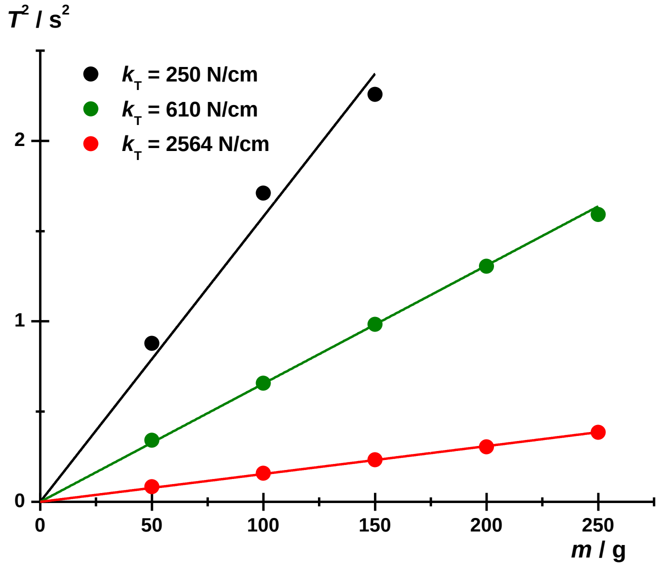

Fig. 4: Square of period of oscillation T^2 as a function of m.

- Plot the spring constants k_T from the dynamic measurements against those obtained from the static measurements k_s and draw a straight line though the points (Fig. 5).

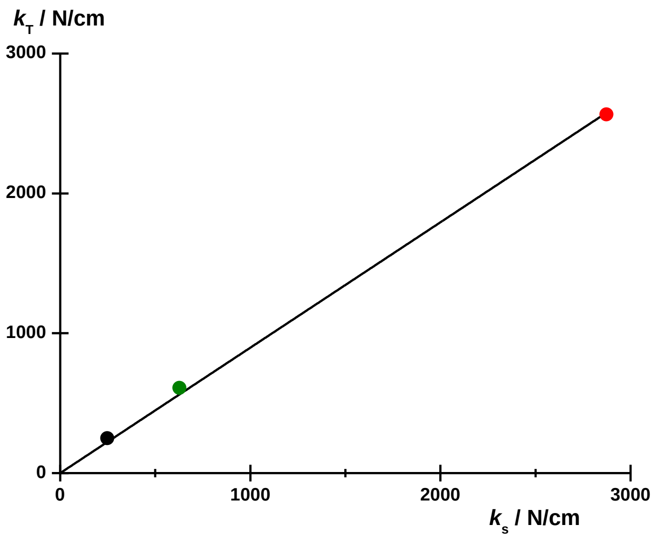

Fig. 5: k_T as a function of k_s with straight line fitted.

Fitting a straight line to the measurements in Fig. 5 results in a line of gradient 0.9, i.e. the values lie along a line bisecting the angle to a good approximation. The constants obtained from both static and dynamic measurements are confirmed to be in agreement.