PHYSICS EXPERIMENT

Mechanical Waves

INVESTIGATE STANDING WAVES ALONG A STRETCHED COIL SPRING AND A TAUT ROPE.

- Generate standing longitudinal waves in a coil spring and standing transverse waves along a rope.
- Measure the intrinsic frequency f_n as a function of number of nodes n.
- Determine the corresponding wavelength λ_n and speed of propagation of the waves c.

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Fig. 1: Measurement set-up for investigating standing waves along a taut rope (left) and a stretched coil spring (right)

GENERAL PRINCIPLES

Some examples of where mechanical waves arise include a stretched coil spring or a taut rope. The waves arising in the spring are longitudinal waves since the deflection of the coil is in the direction of propagation. The waves along a rope by contrast are transverse waves. This is because the incoming wave and the wave reflected at the fixed end have the same amplitude and are superimposed on one another. If the other end is also fixed, the only way that waves can propagate is if resonance conditions are met.

Let $\xi(x,t)$ be the longitudinal or transverse deflection at a point $\underline{x}$ along the carrier medium at a point in time t. The following is then true:

$$(1)\quad \xi_1(x,t) = \xi_0 \cdot \cos\left(2\pi \cdot f \cdot t - \frac{2\pi}{\lambda} \cdot x\right)$$

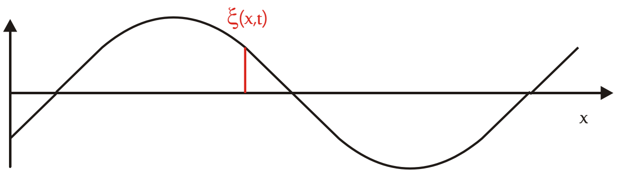

Fig. 2: Illustration of how the localised deflection $\xi(x,t)$ is defined

This applies to a sinusoidal wave travelling from left to right along the carrier medium. The frequency f and wavelength λ are related in following way:

(2) $c = f \cdot \lambda$
c: Propagation velocity of wave

If such a wave, travelling from left to right, should be reflected from a fixed point at $x = 0$, a wave travelling from right to left direction then arises.

(3) $\xi_2(x,t) = -\xi_0 \cdot \cos(2\pi \cdot f \cdot t + \frac{2\pi}{\lambda} \cdot x)$

The two waves are then superimposed to create a standing wave.

(4) $\xi(x,t) = 2\xi_0 \cdot \sin(2\pi \cdot f \cdot t) \cdot \sin(\frac{2\pi}{\lambda} \cdot x)$

These considerations are valid regardless of the nature of the wave or of the carrier medium.

If the other end is also fixed at a position $x = L$, then the following resonance condition needs to be fulfilled at all times t.

(5) $\xi(L,t) = 0 = \sin(\frac{2\pi}{\lambda} \cdot L)$

This only applies if the wavelength meets the following conditions:

(6a) $\frac{2\pi}{\lambda_n} \cdot L = (n+1) \cdot \pi$ bzw. $\lambda_n = 2 \cdot \frac{L}{n+1}$

or $L = (n+1) \cdot \frac{\lambda_n}{2}$

According to equation (2), the frequency is then

(6b) $f_n = (n+1) \cdot \frac{c}{2 \cdot L}$

This implies that the condition for resonance (5) is only fulfilled if the length L is an integer multiple of half the wavelength. The resonant frequency must correspond to this wavelength. In this case, n is the number of nodes in the oscillation. This is zero if there is only one anti-node in the fundamental oscillation (see Fig. 3).

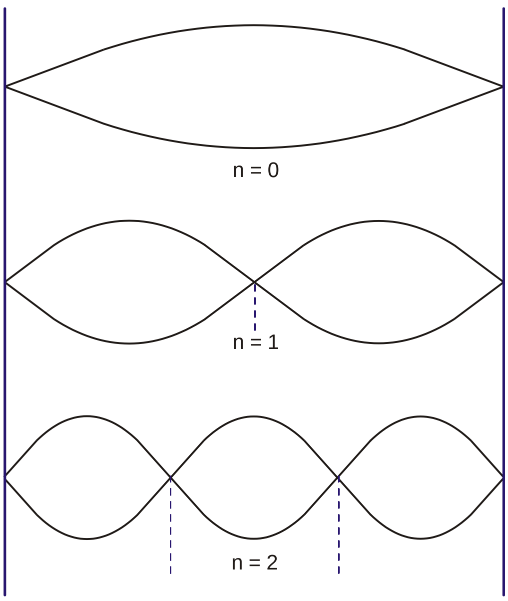

Fig. 3: Standing waves

In this experiment, the carrier medium is either a spring or a rope which is fixed at one end. The other end is connected to a vibration generator at a distance L from this fixed point. This uses a function generator to drive small-amplitude oscillations of variable frequency f. This end can also be regarded as a fixed point to a good approximation.

LIST OF EQUIPMENT

1 Accessories for Spring Oscillations 1000703 (U56003)
1 Accessories for Rope Waves 1008540 (U85560081)
1 Vibration Generator 1000701 (U56001)
1 Function Generator FG 100 @230V 1009957 (U8533600-230)
or
1 Function Generator FG 100 @115V 1009956 (U8533600-115)
1 Precision Dynamometer, 2 N 1003105 (U20033)
1 Pocket Measuring Tape, 2 m 1002603 (U10073)
1 Pair of Safety Experimental Leads, 75cm, red/blue 1017718 (U13816)

SET UP

Waves along a coil spring

- Fasten the angled stand rod into the holder on the rear of the vibration generator.
- Hang one end of the coil spring from the angled stand rod and attach the other end to the pin clamp with the help of the knurled screw.
- Attach the spring to the vibration generator with the help of the pin clamp and stretch it taut by this means.
- Set the (effective) length of the spring L (Fig. 4a) to about 30 cm. You might need to adjust the position of the angled stand rod.
- Connect the function generator to the vibration generator.

Waves on a rope

- Before setting up the apparatus, remove the transport restraint (nut and bolt) from the base plate.
- Screw the short stand rod the base plate. Screw the long stand rod onto the end of the short rod.
- Push the pulley and the holder for the dynamometer onto the stand rod and fasten them there.
- Fasten the stand rod with the pin clamp attached into the holder on the rear of the vibration generator.
- Suspend the dynamometer from the holder. You might need to calibrate the zero position prior to this.
- Suspend the rubber rope for the dynamometer and thread it under the pulley towards the vibration generator. Make sure the rope is as nearly as possible parallel to the table top.
- Thread the rope through the pin clamp on the vibrating stem of the vibration generator and through the pin clamp on the stand rod. First secure the rope to the stand rod with the help of the knurled screw. This relieves the speaker membrane from transverse forces (Fig. 5).

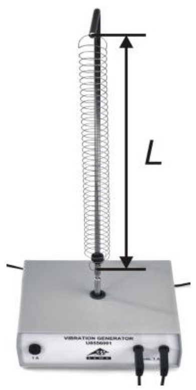

Fig. 4a: Illustration of the (effective) length L of the stretched coil spring.

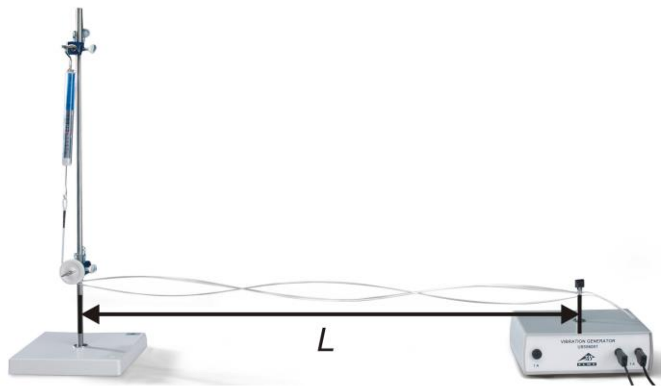

Fig. 4b: Illustration of the (effective) length L of the stretched rope

- Set up the distance from the stand and pulley to the vibration generator in such a way that the (effective) length of the rope L (Fig. 4b) is about 90 cm. Stretch the rope taut with the help of the dynamometer ($F \approx 0.6$ N) but only secure it gently for now with the help of the knurled screw on the vibrating stem.
- Connect the function generator to the vibration generator.

PROCEDURE

- Measure the effective lengths L of the the stretched coil spring and the stretched rope (Fig. 4a, b) and make a note of them.
- Select a “Sine” waveform on the function generator. Set the amplitude adjustment knob to 5 V (12 o’clock setting).
- For both the spring and the rope, increase the frequency of oscillation starting at 1 Hz and slowly rising in steps of 0.1 Hz. Make a note of the resonant frequencies at which no nodes (one anti-node) appear in the waves. Repeat for one node, two nodes three nodes, four nodes and five nodes into tables 1 and 2.
- Increase the forces of tension in the rope to 1.0 N and then to 1.4 N by moving the dynamometer higher up the stand rod. Repeat the measurement in each case and enter the resonant frequencies into Table 2.
- To determine the mass per unit length of the rope directly, measure the total length L_0 and mass m of the rope.

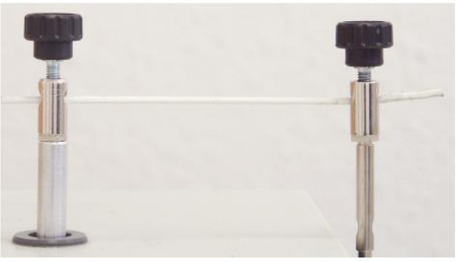

Fig. 5: Illustration of relief for transverse forces on rope.

SAMPLE MEASUREMENT

Length of stretched coil spring L: 0.31 m
Length of stretched rope L: 0.90 m

Tab. 1: Resonant frequency as a function of the number of nodes for waves along a coil spring

n	f_n / Hz
0	7.7
1	15.4
2	23.0
3	30.6
4	38.6
5	45.7

Tab. 2: Resonant frequency as a function of the number of nodes of the standing wave for various forces of tension.

n	f_n / Hz		
	F= 0.6 N	F= 1.0 N	F= 1.4 N
0	7.9	9.8	12.1
1	15.7	19.6	24.0
2	23.4	29.4	35.7
3	30.9	39.2	47.3
4	39.4	49.5	59.2
5	47.5	58.7	71.7

Total length of rope L_0: 1.05 m
Mass of rope m: 3.3 g

EVALUATION

Determining the speed of propagation of the waves *c*

If the resonant frequency is plotted against the number of nodes in the oscillation, equation (6b) indicates that the measured points should lie along a straight line with the´following gradient:

(7) $\alpha = \frac{c}{2 \cdot L} \Leftrightarrow c = 2 \cdot L \cdot \alpha$.

If the length L is known, it is possible to calculate the speed of propagation of the waves c.

- Plot a graph of the resonant frequencies f_n for waves along the coil spring (Table 1) and waves along a rope (Table 2) against the number of nodes n in the oscillation and fit straight lines to the results in each case (Fig. 6, Fig. 7).
- Determine the speeds of propagation c from the gradients α and enter them into Table 3 (waves along spring) and Table 4 (waves along rope).

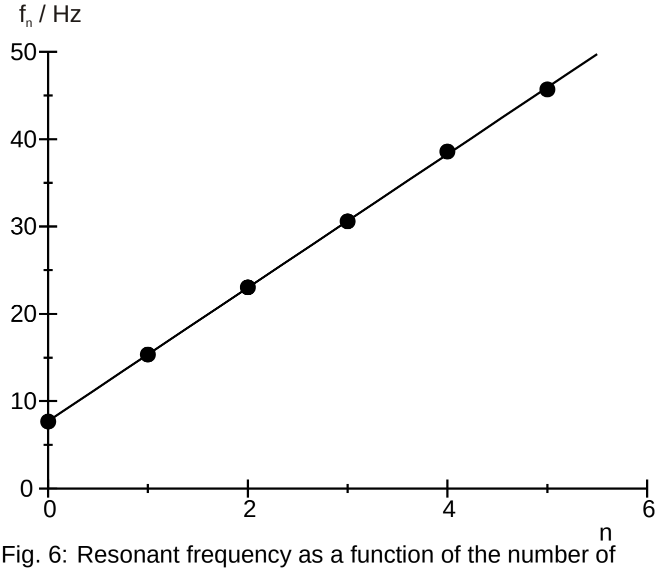

Fig. 6: Resonant frequency as a function of the number of nodes for waves along a coil spring

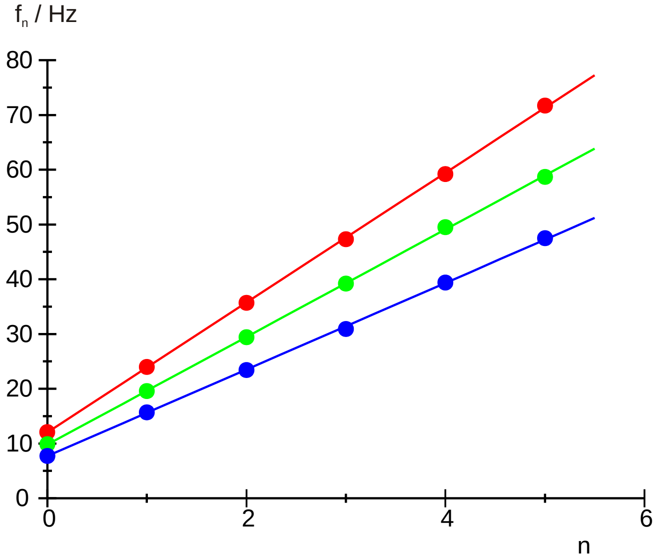

Fig. 7: Resonant frequencies as a function of number of nodes for waves along a rope with forces of tension F = 0.6 N (blue), F = 1.0 N (green) und F = 1.4 N (red).

Tab. 3: Gradient of matching straight line and speeds of propagation as determined therefrom for waves on a coil spring, length of (stretched) spring L = 0.31 m.

α / Hz	c / m/s
7.6	4.7

Tab. 4: Steigung der angepassten Geraden, daraus bestimmte Wellengeschwindigkeiten und deren Quadrate für die Seilwellen bei verschiedenen Spannkräften, Länge des (gespannten) Seils L = 0.90 m.

F / N	α / Hz	c / m/s	c^2 / m²/s²
0.6	7.9	14.2	202
1.0	9.8	17.6	310
1.4	11.9	21.4	458

Determining the wavelengths λ_n corresponding to resonant frequencies f_n

- Determine the wavelengths λ_n first from the lengths L and number of nodes n and then from the resonant frequencies f_n and the speeds of propagation c for waves along the coil (Table 1, Table 3) and along the rope (Table 2, Table 4). Use equations (6a) snd (2) for the calculation and enter the results into table 5 and 6.

Tab. 5: Wavelengths as a function of the number of nodes for waves along a stretched coil spring, length of (stretched) coil spring L = 0.31 m.

n	$\lambda_n = 2 \cdot \frac{L}{n+1}$	$\lambda_n = \frac{c}{f_n}$
0	0.62 m	0.62 m
1	0.31 m	0.31 m
2	0.21 m	0.21 m
3	0.16 m	0.16 m
4	0.12 m	0.12 m
5	0.10 m	0.10 m

Tab. 6: Wavelengths as a function of the number of nodes in waves along a rope, length of (stretched) rope L = 0.90 m.

n	$\lambda_n = 2 \cdot \frac{L}{n+1}$	$\lambda_n = \frac{c}{f_n}$		
		F= 0,6 N	F= 1,0 N	F= 1,4 N
0	1.80 m	1.80 m	1.80 m	1.77 m
1	0.90 m	0.90 m	0.90 m	0.89 m
2	0.60 m	0.61 m	0.60 m	0.60 m
3	0.45 m	0.46 m	0.45 m	0.45 m
4	0.36 m	0.36 m	0.36 m	0.36 m
5	0.30 m	0.30 m	0.30 m	0.30 m

As expected the wavelengths are very well in agreement.

Determining the mass per unit length of the rope μ

If all other parameters are equal, the speed of propagation of the waves on the rope depends on the force of tension F, as indicated by Fig. 7 and Table 4. The following is true:

$$(8) \quad c = \sqrt{\frac{F}{\mu}} \Rightarrow c^2 = \frac{1}{\mu} \cdot F .$$

F: Force of tension
μ :Mass per unit length

- Calculate the squares of the speeds of propagation of the waves c^2 and enter them into Table 4. Plot these values against the force of tension F and fit a straight line to the points (Fig. 8).
- Use equation (8) to obtain the mass per unit length of the rope μ from the gradient by forming the inverse.

$$(9) \quad \mu = \frac{1}{323 \frac{\text{m}}{\text{kg}}} = 0.0031 \frac{\text{kg}}{\text{m}} = 3.10 \frac{\text{g}}{\text{m}} .$$

- Determine the mass per unit length directly by measuring the length and mass of a piece of rope.

$$(10) \quad \mu = \frac{m}{L_0} = \frac{3.3 \text{ g}}{1.05 \text{ m}} = 3.14 \frac{\text{g}}{\text{m}} .$$

The values for the mass per unit length are in agreement to within about 1%.

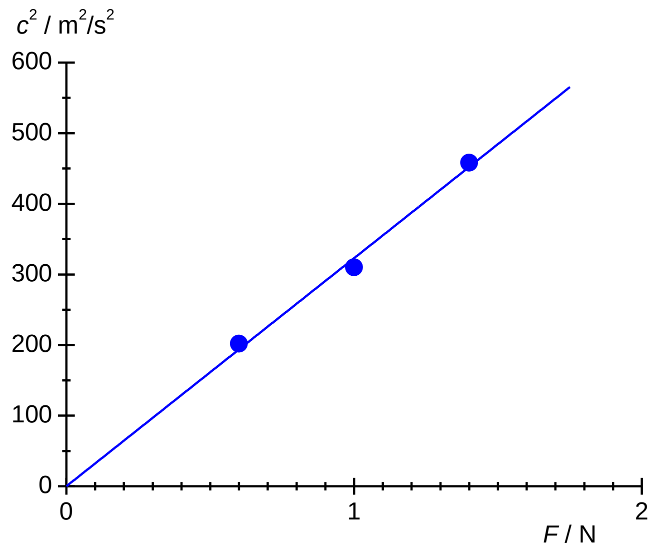

Fig. 8: Square of wave propagation speed c^2 for waves on a rope as a function of F.